

## TEMPERATURE DEPENDENCE OF THE ELASTIC CONSTANTS OF SODIUM

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**Abstract**—The adiabatic elastic constants of single crystal sodium were measured as a function of temperature from 195°K to 78°K by the ultrasonic pulse-echo technique. In order to eliminate transit time errors a buffer technique was employed. The measured elastic constants at 78°K (in units of  $10^{10}$  dyn-cm<sup>-2</sup>) are as follows:

$$C = 5.78, \quad C' = 0.688, \quad C_n = 13.30, \quad \text{and} \quad B_s = 7.30$$

The notation  $C = C_{44}$ ,  $C' = (C_{11} - C_{12})/2$ ,  $C_n = (C_{11} + C_{12} + 2C_{44})/2$ , and  $B_s = C_n - C - C'/3$  is used. The shear constants  $C$  and  $C'$  are interpreted in terms of Fuchs' electrostatic contribution to the shear stiffness of the alkali metals. The results of this work are in agreement with those of Daniels at 300°K and the shear constants are within 3% of the values computed by Quimby and Siegel. The results are also compared with Bender's value of the elastic constants at 90°K and with Swenson's isothermal compressional data.

### INTRODUCTION

THE single crystal elastic shear constants of sodium were first determined by QUIMBY and SIEGEL.<sup>(1)</sup> They employed a composite oscillator technique over the temperature range 80°–210°K. O. BENDER<sup>(2)</sup> made static measurements on single crystals of sodium at 90°K. In both investigations the elastic compliances were the directly measured quantities and from this data the elastic constants were computed. Since a 15% discrepancy exists in these data and since the elastic constants were not directly determined, it was decided to measure the elastic constants directly by the conventional and more reliable ultrasonic pulse-echo techniques over the temperature range 78°–195°K.

FUCHS<sup>(3)</sup> has calculated the theoretical values of the elastic shear stiffnesses for the alkali metals at absolute zero. He predicts that the main contribution of the shear stiffnesses in these metals arises from the long range electrostatic interaction between ion cores and the valence electrons, which accounts for the large elastic anisotropy observed in the alkali metals.<sup>(4-7)</sup> He also includes a second,

but minor, constituent arising from short range ion core interactions. Since the ultrasonic pulse-echo technique yields directly measured values for the shear constants,  $C = C_{44}$  and  $C' = (C_{11} - C_{12})/2$ , a direct comparison with Fuchs' theoretical values of these constants can be made.

### EXPERIMENTAL PROCEDURE

The procedure for crystal growth, orientation, and preparation of the acoustic specimens was the same as described in a paper by DANIELS<sup>(8)</sup> and similar to the techniques employed on potassium presented in a recent publication from this laboratory.<sup>(7)</sup>

For a cubic material all three independent elastic constants can be obtained by the ultrasonic pulse-echo technique if the appropriate wave is propagated along the [110] direction. Generation of longitudinal, fast shear, and slow shear waves was straightforward once the proper handling techniques were perfected. Because of the high anisotropy of sodium, generation of slow shear waves caused the most difficulty and proved quite sensitive to crystal orientation and to proper positioning of the polarization direction of the Y-cut transducer. Two crystals oriented in the [110] direction

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were used to obtain elastic constant measurements as a function of temperature. A third crystal oriented along [100] was used to measure the elastic constant  $C_{11}$  directly.

The elastic constants were measured by a buffer technique.<sup>(7)</sup> Adoption of the buffer technique eliminates both the transit time error and the necessity of applying a transducer directly on the soft metal. According to this method the transducer and sample are cemented at opposite ends of a buffer rod. A pulse propagated through the rod is partially reflected and partially transmitted at the buffer-specimen interface. The transit time measured is then the true transit time. In this work magnesium served quite well as the buffer due to the desirable acoustic impedance reflection and transmission coefficients which exist between a magnesium-sodium interface.

A silicone release agent, Dow Corning 7 Compound, was used as a bond between both the quartz and magnesium and the magnesium and sodium. This seal, which does not react with sodium, was fluid at temperatures below that of the melting point of sodium so that thin seals could be made without introducing distortions, glassy at low temperatures to compensate for the relative thermal expansion differences involved, and rigid enough to provide good acoustic coupling for shear waves at the temperatures desired.

Transit time measurements for all three wave propagations were taken at liquid nitrogen and dry ice temperatures. In addition, measurements were taken at 20° temperature intervals between 78°K and 195°K. These intermediate temperatures were determined by placing an iron-constantan thermocouple in good thermal contact with the aluminum cartridge which contained the buffer assembly. About 2 hr were required to go from 78° to 195°K. The round trip transit times measured were of the order of 6, 10 and 28  $\mu$ sec for longitudinal, fast shear, and slow shear waves respectively for a 1 cm sample.

At 295°K sample 1 had a length of 1.34 cm and sample 2 a length of 1.05 cm. The sample lengths were measured at room temperature with a micrometer and corrections appropriate to the measuring temperature were made using the thermal expansion data of QUIMBY and SIEGEL.<sup>(9)</sup>

The elastic constants were calculated from the density, transit time, and sample length and plotted

as a function of temperature. From these graphs a set of elastic constants were chosen for each 20°K temperature interval. The temperatures selected were those for which a complete set of measurements as described was obtained for the two [110] samples. In addition, new seals were placed on each of these crystals and the measurements repeated. The velocities obtained in various runs always agreed within 1%. A third crystal was measured at 78°K and the results also agreed with the data reported here. In addition, longitudinal velocities were measured as a function of temperature on a crystal oriented along [100] and are reported in the next section.

## RESULTS

The elastic constants are related to the acoustic wave velocities in the [110] direction by the following equations:

$$\begin{aligned} C_n &= (C_{11} + C_{12} + 2C_{44})/2 = \rho v_2^2 \\ C &= C_{44} = \rho v_5^2 \\ C' &= (C_{11} - C_{12})/2 = \rho v_4^2 \end{aligned} \quad (1)$$

where  $\rho$  is the density,  $v_2$  is the longitudinal velocity,  $v_5$  is the fast shear velocity with particle motion in the [001] direction, and  $v_4$  is the slow shear velocity with particle motion in the [110] direction. The adiabatic bulk modulus,  $B_s$ , and the stiffness  $C_{11}$  and  $C_{12}$  can be obtained by combining the directly measured quantities in the following way:

$$\begin{aligned} B_s &= \rho v_2^2 - \rho v_5^2 - \rho v_4^2/3 = (C_{11} + 2C_{12})/3 \\ C_{11} &= \rho v_2^2 - \rho v_5^2 + \rho v_4^2 \\ C_{12} &= \rho v_2^2 - \rho v_5^2 - \rho v_4^2 \end{aligned} \quad (2)$$

The elastic constants shown in Table 1 have been calculated using equations (1) and (2).

Table 2 lists the adiabatic bulk modulus  $B_s$ , along with the isothermal bulk modulus  $B_T$ . The latter was computed from  $B_s$  using the approximate relation

$$B_T \approx B_s \left( 1 - \frac{TV\beta^2 B_s}{C_p} \right)$$

where  $V$  is the molar volume,  $\beta$  is the volume coefficient of expansion and  $C_p$  is the molar heat capacity. The values of  $V$  and  $\beta$  were obtained